THE SYNTHESIS OF TERNARY FUNCTIONS UNDER FIXED POLARITIES AND TERNARY I^2L CIRCUITS

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Abstract

This paper discusses the expansion of multiple-value functions based upon modulo-algebra and Kronecker product. A transform algorithm of the expansion coefficients of various polarities, and their minimisation are proposed. Ternary function realizations using I^2L technology are finally considered.

List of Symbols

n: number of independent variables

$x_i, i = 0$ to $n-1, x_i \in \{0,1,2\}$: independent input variables

$f(x_{n-1},\ldots,x_1,x_0)$, abbreviated to $f(x)$: function of the x_i input variables, $f(x) \in \{0,1,2\}$

$x_i + x_j$: arithmetic sum of x_i and x_j

$x_i \oplus x_j$: mod-3 addition of x_i and x_j

$x_i \cdot x_j$: mod-3 multiplication of x_i and x_j

$\dot{x}_i, i = 0$ to $n-1$: various polarities of variable x_i

$\tilde{x}_i$: $= x_i \oplus 1$

$\tilde{\tilde{x}}_i$: $= x_i \oplus 2$

$\bar{x}_i^B$: complement operation on variable x_i,

$$\bar{x}_i^B = \begin{cases} B - x_i \text{ if } B > x_i \\ 0 \text{ otherwise,} \end{cases}$$

$x_i \wedge x_j$: minimum of x_i and x_j

$x_i \vee x_j$: maximum of x_i and x_j

$\overline{x_i \vee x_j}^B$: complement of maximum of x_i and x_j,

$$\overline{x_i \vee x_j}^B = \begin{cases} B - (x_i \vee x_j) \text{ if } B > x_i \vee x_j \\ 0 \text{ otherwise} \end{cases}$$

$U_f(\alpha,\beta,\gamma,x)$: Universal-logic-module for ternary functions, based upon Reed-Muller expansion, i.e. modulo-algebra expansion

$U_h(\xi,\eta,x)$: basic Universal-logic-module for ternary functions, based upon modulo-algebra expansion $U_h(\xi,\eta,x) = \xi \oplus \eta x$

F]: column vector whose entries are the 3^n values of $f(x)$, arranged in increasing order of y, $y = \sum_{i=0}^{n-1} x_i 3^i$

$L_B(K)]$: coefficient column vector based upon modulo-algebra expansion under the polarity $k_{n-1}\ell_{n-1}\ldots k_1\ell_1 k_0\ell_0$, where K,L are the decimal expressions of $k_{n-1}\ldots k_1k_0$, $\ell_{n-1}\ldots \ell_1\ell_0$, respectively

[T]: transform matrix from B] to F]

$[T]^{-1}$: transform matrix from F] to B]

$\otimes$: Kronecker product

$\bigotimes_{i=1}^{n-1} [A_i]$: $= [A_{n-1}] \otimes \ldots\ldots \otimes [A_1] \otimes [A_0]$

$[P_k]$: transform matrix of expansion coefficients when $x \to x \oplus k$

$[{}^{\ell}I]$: transform matrix of expansion coefficients when $x \to (\ell + 1)x$

[P].[T]: product of matrices over GF(3)

Introduction

Several modulo-algebra expansions of multiple-valued functions have been proposed [1]-[3]. Lately the modulo-algebra expansion of multiple-valued functions over Galois field with q numbers, where q is a prime number or a power of a prime number, has been investigated [4]. We will discuss the transform between F] and the Reed-Muller expansion coefficient column matrix B], and also among various B(K)] under different polarities in section 2 by means of the Kronecker product. Minimal modulo-algebra expansion for ternary functions under fixed polarities is further considered.

According to the modulo-algebra expansion, a Universal-logic-module U_f can be introduced [5], where

$$U_f(\alpha,\beta,\gamma,x) = \alpha \oplus \beta x \oplus \gamma x^2 \quad (1)$$

It has been shown that any ternary function may be realized by the use of only this kind of module. Here in section 3 it will be shown that such a Universal-logic-module $U_f(\alpha,\beta,\gamma,x)$ can be composed of two basic Universal-logic-modules $U_h(\xi,\eta,x)$, where $U_h(\xi,\eta,x) = \xi \oplus \eta x$.

The realization of multiple-valued functions using I^2L circuits can be found published [6],[7]. In section 4 a number of further I^2L circuits realizing ternary functions are presented.

Modulo-algebra Expansion of Multiple-Valued functions under fixed polarities

It is well known that a single-variable ternary function can be written as:

$$f(x) = b_0 \oplus b_1 x \oplus b_2 x^2$$

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Define F] as a column vector of a function f, as previously defined. Define B] as the Reed-Muller coefficient column vector for f based upon modulo-algebra expansion. Define [T] as transform matrix from B] to F].

For a single variable, we have:

$$F] = f_0 f_1 f_2]^t \tag{2}$$

where f_0, f_1, f_2 correspond to f(0),f(1),f(2) and

$$\left.\begin{aligned} f(0) &= b_0, \\ f(1) &= b_0 \oplus b_1 \oplus b_2, \\ f(2) &= b_0 \oplus 2b_1 \oplus b_2 \end{aligned}\right\} \tag{3}$$

$$B] = b_0 b_1 b_2]^t \tag{4}$$

$$F] = [T].B] = \begin{bmatrix} T_{00} & T_{01} & T_{02} \\ T_{10} & T_{11} & T_{12} \\ T_{20} & T_{21} & T_{22} \end{bmatrix} . \begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix} \tag{5}$$

Substituting equation (3) into (2), and comparing the result with (5), the transform matrix may be obtained:

$$[T] = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix} \tag{6}$$

The inverse transform $[T]^{-1}$ is termed transform matrix from F] to B]:

$$B] = [T]^{-1} F] \tag{7}$$

Similarly, $[T]^{-1}$ can be determined as follows:

$$[T]^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 2 & 2 & 2 \end{bmatrix} \tag{8}$$

Note, both [T] and $[T]^{-1}$ here are over a single variable. For a two-variable ternary function, we have:

$$f(x_1, x_0) = b_0 \oplus b_1 x_0 \oplus b_2 x_0^2 \oplus b_3 x_1 \oplus b_4 x_1 x_0$$
$$\oplus\, b_5 x_1 x_0^2 \oplus b_6 x_1^2 \oplus b_7 x_1^2 x_0 \oplus b_8 x_1^2 x_0^2 \tag{9}$$

In accordance with the Kronecker product and its properties, the above equation may be simplified to:

$$f(x_1, x_0) = [(1 x_1 x_1^2) \otimes (1 x_0 x_0^2)].B], \tag{10}$$

where $\otimes$ denotes the Kronecker product. It may be proved that there are the following relations:

$$F] = [T]^{\otimes 2} .B] \tag{11}$$

and $$B] = \left\{[T]^{-1}\right\}^{\otimes 2} .F] \tag{12}$$

where

$$[T]^{\otimes 2} = [T] \otimes [T] = \begin{bmatrix} 1&0&0 & 0&0&0 & 0&0&0 \\ 1&1&1 & 0&0&0 & 0&0&0 \\ 1&2&1 & 0&0&0 & 0&0&0 \\ 1&0&0 & 1&0&0 & 1&0&0 \\ 1&1&1 & 1&1&1 & 1&1&1 \\ 1&2&1 & 1&2&1 & 1&2&1 \\ 1&0&0 & 2&0&0 & 1&0&0 \\ 1&1&1 & 2&2&2 & 1&1&1 \\ 1&2&1 & 2&1&2 & 1&2&1 \end{bmatrix} \tag{13}$$

$$\left\{[T]^{-1}\right\}^{\otimes 2} = [T]^{-1} \otimes [T]^{-1} = \begin{bmatrix} 1&0&0 & 0&0&0 & 0&0&0 \\ 0&2&1 & 0&0&0 & 0&0&0 \\ 2&2&2 & 0&0&0 & 0&0&0 \\ 0&0&0 & 2&0&0 & 1&0&0 \\ 0&0&0 & 0&1&2 & 0&2&1 \\ 0&0&0 & 1&1&1 & 2&2&2 \\ 2&0&0 & 2&0&0 & 2&0&0 \\ 0&1&2 & 0&1&2 & 0&1&2 \\ 1&1&1 & 1&1&1 & 1&1&1 \end{bmatrix} \tag{14}$$

Similarly for n-variable ternary functions, the following equations may be derived:

$$f(x_{n-1}, \ldots, x_1, x_0) = \left[\bigotimes_{i=0}^{n-1} (1 x_i x_i^2)\right].B \tag{15}$$

$$F] = [T]^{\otimes n} .B] \tag{16}$$

$$B] = \left\{[T]^{-1}\right\}^{\otimes n} .F] \tag{17}$$

Recalling that the Reed-Muller expansion of a binary function may be derived under different polarities of the input variables, the concept of variable -polarity can be introduced in the multiple-valued function case as well [8,9]. Consider the generalized modulo-algebra expansion of an n-variable ternary function:

$$f(\dot{x}_{n-1} \ldots, \dot{x}_1, \dot{x}_0) = \left[\bigotimes_{i=0}^{n-1} (1 \dot{x}_i \dot{x}_i^2)\right].B],$$
$$= b_0 \oplus b_1 \dot{x}_0 \oplus b_2 \dot{x}_0^2 \oplus \ldots .$$
$$\oplus\, b_{3^n-1} \dot{x}_{n-1}^2 \ldots \dot{x}_1^2 \dot{x}_0^2, \tag{18}$$

where $\dot{x}$ is defined as the polarity-expression of x, where it takes a different output value for each possible input value, i.e. there is no loss of information. In ternary system each variable may take six possible polarities, including the variable value itself. They are shown in Table 1.

In the above table the entries in the left column are corresponding modulo-algebra expansions of various polarity-expressions of variable x_i. Note that the last three entries are mod-3 products of the corresponding above entries and constant two. Thus a generalized equation can be obtained:

$$\dot{x}_i = (\ell \oplus 1).(x_i \oplus k), \tag{19}$$

	x_i	0	1	2
$\dot{x}_i$	$1 \oplus x_i$	1	2	0
	$2 \oplus x_i$	2	0	1
	$2x_i$	0	2	1
	$2 \oplus 2x_i$	2	1	0
	$1 \oplus 2x_i$	1	0	2

Table 1 The polarities of a ternary variable x_i

$k = 0,1,2$ and $\ell = 0,1$

Therefore there are $(3!)^n = 6^n$ possible n-variable ternary modulo-algebra expansions. It is said to be the unmodified form when:

$\ell = 0,\ k = 0$

The above transform (19) can be divided into two steps. The first transform is termed the k-transform, in which we consider $\ell = 0$. The second transform is termed the ℓ-transform. Now let us consider the k-transform. Take a single-variable function as an example.

Suppose

$$f(x) = b_0^{(k)} \oplus b_1^{(k)}.\dot{x} \oplus b_2^{(k)}.\dot{x}^2 \qquad$$
$$= [1\dot{x}\dot{x}^2].[b_0^{(k)} b_1^{(k)} b_2^{(k)}]^t \qquad (20)$$

Consider the k-transform

$$\dot{x} = x \oplus k,\ k = 0,1,2 \qquad (21)$$

If $b_i^{(0)}$ is expressed by b_i, i.e. $b_i \equiv b_i^{(0)}$, then the standard form ($k = 0$, $\ell = 0$) may be obtained:

$$f(x) = b_0 \oplus b_1 x \oplus b_2 x^2 = \left[1xx^2\right]. \begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix} \qquad (22)$$

From (21) we obtain:

$$(1\dot{x}\dot{x}^2) = (1xx^2). \begin{bmatrix} 1 & k & k^2 \\ 0 & 1 & 2k \\ 0 & 0 & 1 \end{bmatrix} \qquad (23)$$

and

$$(1xx^2) = (1\dot{x}\dot{x}^2). \begin{bmatrix} 1 & 2k & k^2 \\ 0 & 1 & k \\ 0 & 0 & 1 \end{bmatrix} \qquad (24)$$

If we define

$$[P_k] = \begin{bmatrix} 1 & 2k & k^2 \\ 0 & 1 & k \\ 0 & 0 & 1 \end{bmatrix} \qquad (25)$$

and

$$[P_k]^{-1} = \begin{bmatrix} 1 & k & k^2 \\ 0 & 1 & 2k \\ 0 & 0 & 1 \end{bmatrix} \qquad (26)$$

Substituting (24) into (22) and comparing with (20), we obtain:

$$B^{(K)}] = [P_k].B]$$

Similarly the reverse transform is as follows:

$$B] = [P_k]^{-1}.B^{(K)}]$$

For a n-variable ternary system, 3^n different modulo-algebra expansion coefficient vectors $B^{(K)}]$ can be derived if each variable $\dot{x}_i = x_i \oplus k_i$ takes every possible value, where K is decimal number of ternary $k_{n-1}\ldots k_1 k_0$, $K = 0,1,\ldots 3^n-1$.

The following equation may be obtained by using a Kronecker product and the equation (25):

$$B^{(K)}] = \left\{ \bigotimes_{i=0}^{n-1} [P_{k_i}] \right\}.B] \qquad (27)$$

It is obvious that the complexity of the various modulo-algebra expansion coefficient vectors $B^{(K)}]$ of a function varies with K. The more zero-coefficients in $B^{(K)}]$, i.e. the fewer the necessary product-terms in the modulo-algebra expansion, the simpler the function form.

Example 1

Consider a two-variable ternary function expressed by Fig.1. From the K-map its column vector is

$$F] = 0\ 2\ 0\ 2\ 1\ 0\ 0\ 2\ 0]^t$$

$x_0 \backslash x_1$	0	1	2
0	0	2	0
1	2	1	2
2	0	0	0

Fig.1 Example 2-variable ternary function

The unmodified expansion coefficient vector can be obtained from the equations (12) and (14):

$$B] = 0\ 1\ 1\ 1\ 2\ 1\ 1\ 2\ 1]^t$$

Therefore its modulo-algebra expansion is as follows:

$$f(x_1,x_0) = x_0 \oplus x_0^2 \oplus x_1 \oplus 2x_1x_0 \oplus x_1x_0^2$$
$$\oplus x_1^2 \oplus 2x_1^2x_0 \oplus x_1^2x_0^2$$

When the eight other possible polarities are considered, it is found that there are the least non-zero coefficients when $k_1 = 2$, $k_0 = 1$, i.e. $K = 7$.

The corresponding expansion coefficient vector can be derived from the equations (25) and (27):

$$B^{(7)}] = ([P_2] \otimes [P_1]).B] = \left[\begin{bmatrix}1&1&1\\0&1&2\\0&0&1\end{bmatrix} \otimes \begin{bmatrix}1&2&1\\0&1&1\\0&0&1\end{bmatrix}\right].B]$$

$$\begin{bmatrix}1&2&1&1&2&1&1&2&1\\0&1&1&0&1&1&0&1&1\\0&0&1&0&0&1&0&0&1\\0&0&0&1&2&1&2&1&2\\0&0&0&0&1&1&0&2&2\\0&0&0&0&0&1&0&0&2\\0&0&0&0&0&0&1&2&1\\0&0&0&0&0&0&0&1&1\\0&0&0&0&0&0&0&0&1\end{bmatrix}\begin{bmatrix}0\\1\\1\\1\\2\\1\\1\\2\\1\end{bmatrix} = \begin{bmatrix}0\\2\\0\\0\\0\\0\\0\\0\\1\end{bmatrix}$$

Its corresponding modulo-algebra expansion therefore is as follows:

$$f(x_1,x_0) = 2\tilde{x}_0 \oplus \tilde{\tilde{x}}_1^2.\tilde{x}_0^2 .$$

After choosing the optimum K, further consider the determination of optimum L. where L is the decimal expression of binary $\ell_{n-1}\cdots\ell_0$, $\ell_i \in (0,1)$. Similarly, we may obtain transforms corresponding to a replacement of $\tilde{x} = (\ell \oplus 1).x$. For a single-variable ternary function, we find:

$$\left.\begin{aligned} B^{(K)}] &= [^{\ell}I].^{\ell}B^{(K)}] \\ ^{\ell}B^{(K)}] &= [^{\ell}I]^{-1}.B^{(K)}] \end{aligned}\right\} \quad (28)$$

where

$$[^{\ell}I] = [^{\ell}I]^{-1} = \begin{bmatrix}1&0&0\\0&1+\ell&0\\0&0&1\end{bmatrix} \quad (29)$$

It can be seen that the ℓ-transform does not change the number of non-zero coefficients, since all diagonal elements are non-zero and the others are zero in $[^{\ell}I]$. For n-variable ternary function, we derive

$$\left.\begin{aligned} ^{L}B^{(K)}] &= \left\{\bigotimes_{i=0}^{n-1}\left[^{\ell_i}I\right]\right\}.B^{(K)}] \\ B^{(K)}] &= \left\{\bigotimes_{i=0}^{n-1}\left[^{\ell_i}I\right]\right\}.^{L}B^{(K)}] \end{aligned}\right\} \quad (30)$$

Although the number of non-zero coefficients remains unchanged, the number of coefficients with value 2 varies with different L. The fewer the number of these coefficients, the simpler is the corresponding modulo-algebra expansion of a function. Therefore the optimum procedure can be stated as follows. Firstly search for the optimal K value under the K-transform so that the number of non-zero coefficients is minimised. Then determine the optimal L value under L-transform so that the number of coefficients being two is minimum. It may be seen that only $3^n + 2^n$ searches are necessary for any n-variable ternary function, against 6^n exhaustive searches. Consider the above Example 1, we can find the number of coefficients is being two is minimum when $L = 1$, i.e. $\ell_1 = 0$, $\ell_0 = 1$. The corresponding coefficient vector is:

$$L_B(K)] = 1_B(7)] = 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 1]^t$$

The corresponding modulo-algebra expansion is:

$$f(x_1,x_0) = (2\tilde{x}_0) \oplus (\tilde{\tilde{x}}_1)^2.(2\tilde{x}_0)^2$$

Note that in this case the non-zero coefficients of all products are of unity value.

Universal-Logic-Modules

Hurst and Tokmen have disclosed a Universal-logic-module based on modulo-algebra for a ternary system [5]:

$$U_f(\alpha,\beta,\gamma,x) = \alpha \oplus \beta x \oplus \gamma x^2 \quad (31)$$

This may be decomposed into smaller cells:

$$U_f(\alpha,\beta,\gamma,x) = \alpha \oplus \beta x \oplus \gamma x^2 = \alpha \oplus x(\beta \oplus \gamma x) = \alpha \oplus U_h x$$

$$\text{Here} \quad U_h(\xi,\eta,x) = \xi \oplus \eta x \quad (32)$$

It is obvious that a $U_f(\alpha,\beta,\gamma,x)$ can be realized by two U_h cells as shown in Fig.2. Therefore U_h forms a complete set. The advantages using U_h are a simpler algebraic expression and flexibility in employment. After examining all twenty-seven single variable ternary functions, it can be shown that in addition to constants 0, 1 and 2 and variable x itself, nine of them may be realized by only one U_h. They are:

$f_1(x) = x^2$, (0,x) $\quad$ $f_4(x) = x \oplus x^2$, (x,x)

$f_6(x) = 2x$, (0,2) $\quad$ $f_{10}(x) = 1 \oplus x^2$, (1,x)

$f_{12}(x) = 1 \oplus x$, (1,1) $\quad$ $f_{15}(x) = 1 \oplus 2x$, (1,2)

$f_{19}(x) = 2 \oplus x^2$, (2,x) $\quad$ $f_{21}(x) = 2 \oplus x$, (2,1)

$f_{24}(x) = 2 \oplus 2x$, (2,2)

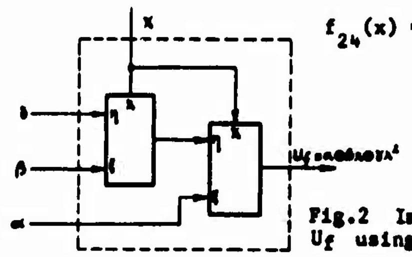

Fig.2 Implementation of U_f using two U_h modules

The values in brackets express the corresponding input patterns. Note five of nine are merely polarities variation of variable x. They are

$$f_{12}(x) = 1 \oplus x, = \tilde{x},\ f_{21}(x) = 2 \oplus x, = \tilde{\tilde{x}},\ f_{6}(x) = 2x,$$

$$f_{24}(x) = 2 \oplus 2x, = 2\tilde{x},\ f_{15}(x) = 1 \oplus 2x, = 2\tilde{\tilde{x}}.$$

Thus the various polarities of variable x can be realized using a single U_h.

Realization of ternary logic using I^2L circuits

The basic operations in modulo-algebra are mod-3 addition, and mod-3 multiplication [10]. They may be reduced to addition, multiplication and mod-3 limit. Here we consider the realization of some ternary logic circuits.

Polarity-transform circuits

Figs.3(a)-(f) give various polarity-transform circuits. Fig.3(a) is a complement circuit; Fig.3(b) is a circuit for multiplication by two. Fig.3(c)-(f) show how other polarity-transform circuits may be obtained in terms of appropriate serial connection of the above two circuits. The function of Figs. 3(e) and (f) is the same, but the circuit of (e) is simpler.

Mod-3 addition

Mod-3 adder is shown in Fig.4. It can be seen that the upper branch creates arithmetic addition, the lower branch being a mod-3 limiter.

Mod-3 multiplication

The difficulty of realizing multiplication of two variables consists in the physical interpretation of multiplicand and multiplier. If product and

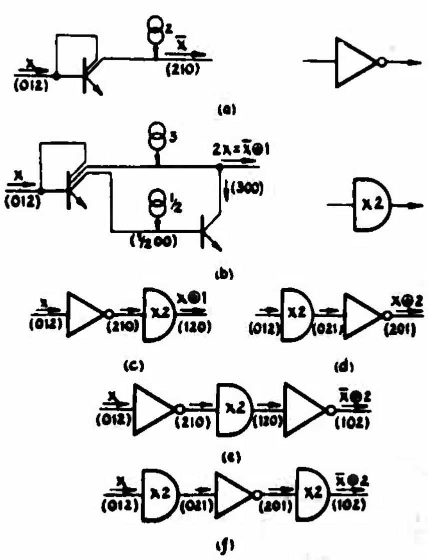

Fig.3 I^2L polarity-transform circuits

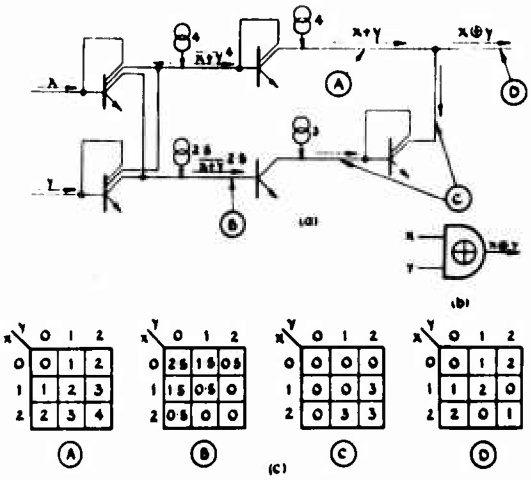

Fig.4 Mod-3 adder realisation
(a) circuit, (b) legend, (c) K-maps

multiplicand are expressed by using the same physical measure, say electrical current, then the multiplier becomes a pure number without any physical meaning. However, up to now any simple effective control of amplification in terms of current has not been found. In a binary system, this difficulty is avoided because the operation of multiplication and the minimum of two variables 0,1 is the same.

We extend this interpretation into the ternary system. Fig. 5(a) illustrates that the multiplication of two variables can be divided into two operations. The central K-map of Fig.5(a) denotes minimum of variables, i.e. $x \wedge y$. Since $x \wedge y = \overline{\bar{x} \vee \bar{y}}$, this may be implemented as shown in Fig.5(b). The RH K-map expresses $\overline{2}(x+y)$, i.e.

$$\overline{2}(x+y) = \begin{cases} x + y - 2, & \text{if } x + y \geq 2 \\ 0 & \text{otherwise} \end{cases}$$

Fig.5(b) gives the total realization. The upper part implements $x \wedge y$; the centre realizes $\overline{2}(x+y)$, and the lower is the mod-3 limiter. The functions of various points A - D are expressed in the corresponding K-maps shown in Fig.5(d).

Arithmetic circuits

On the bases of mod-3 addition and mod-3 multiplication, full-adder and full-multiplier with carry may be designed. Figs.6(a) and (b) show their I^2L circuits respectively, where C is carry input and C' is carry output.

U_h circuit

This may be realized by the cascade of a mod-3 multiplier and a mod-3 adder, as shown in Fig.7.

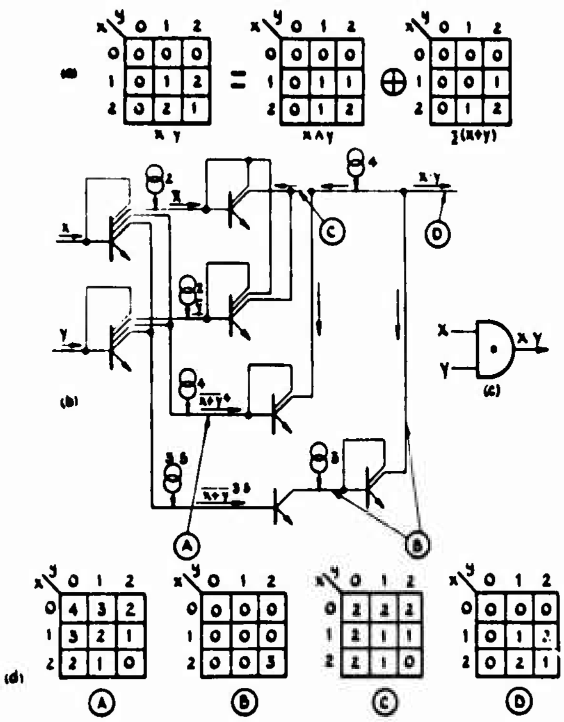

Fig.5 Mod-3 multiplier realisation

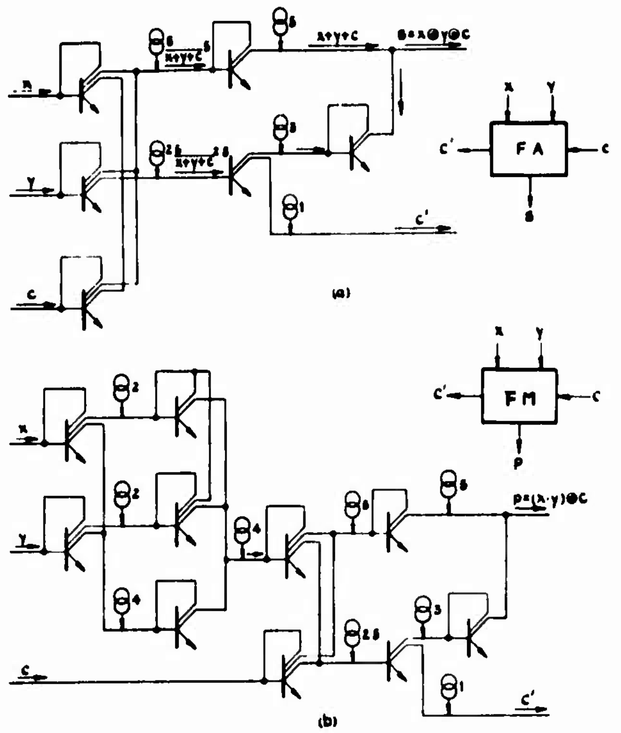

Fig.6 Full-adder and full-multiplier realisations (a) adder, (b) multiplier

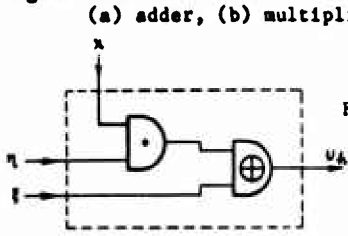

Fig.7 Basic universal-logic-module U_h

Conclusions

The modulo-algebra expansion of a ternary function has been discussed in terms of modulo-algebra and Kronecker product. Because of the six possible polarities of a ternary variable the minimization of ternary functions is more complicated than that of the corresponding binary case. A minimization procedure has been suggested. Since the mod-3 multiplier costs are high, the first step is a search for the optimum K to make the number of non-zero expansion coefficients minimum. On the basis of this new function, we then find the optimum L corresponding to the minimum number of product terms which have coefficients of two. Such a searching process can be implemented by using a computer search.

A basic Universal-logic-module, two of which are capable of realizing any single-variable ternary function, has also been considered. A number of I^2L circuits realizing various ternary functions have been proposed.

References

1. Berlin, R.D., "Synthesis of N-valued switching circuits", IRE Trans., EC-7, 1958, pp.52-56.

2. Lowenschuss, O., "Non-binary switching theory", IRe Natn. Conv. Rec. 6(4), 1958, pp.305-317.

3. Tamari, D., "Some mutual applications of logic and mathematics", Proc. 2nd Int. Colloq. of Mathematical Logic, 1952, pp.89-90.

4. Green, D.H. and Taylor, I.S., "Modular representation of multiple-valued logic systems", Proc.IEE, 121, 1974, pp.409-418.

5. Tokmen, V.H. and Hurst, S.L., "A consideration of Universal-logic-module for ternary synthesis, based upon Reed-Muller coefficients", Proc. 9th Int. Symp. MVL, 1979, pp.248-256.

6. Dao, T.T., McCluskey, E.J. and Russell, L.K., "Multi-valued integrated injection logic", IEEE Trans., C-26, 1977, pp.1233-1241.

7. McCluskey, E.J., "Logic design of multi-valued I^2L logic circuits", IEEE Trans., C-28, 1979, pp.546-559.

8. Wu, X., Chen, X. and Hurst, S.L., "Mapping of Reed-Muller coefficients and the minimisation of exclusive-OR switching functions", Proc.IEE Part E, 129, 1982, pp.15-20.

9. Green, D.H. and Taylor, I.S., "Multiple-valued switching circuit design by means of generalised Reed-Muller expansions", Digital Processes, 2, 1976, pp.63-81.

10. Hurst, S.L., "Logical Processing of Digital Signals", Crane Russak, N.Y. and Edward Arnold, London, 1978.